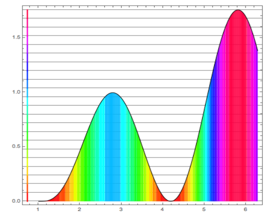
UC RIVERSIDE
Department of Mathematics

MATH 151C - Advanced Calculus



Homework 1

Due date: Tuesday, April 13 at 11:59pm

Instructions:

* You are encourage to work in group but you have to write your individual solution
* You can use any book, article or web-based mathematical material or computational software
* You can ask Ryan, or Estela questions
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The problems from the textbook will be indicated by the chapter and corresponding number.
* The homework needs to be typeset in “LaTeX” .

1. (Chapter 1, Problem 2) Let A be a countable set of real numbers. Use the definition of outer measure to show $m^*(A) = 0$.

2. (Chapter 1, Problem 3) Let S and T be coverings of a set A by intervals.

   a) Explain why $S \cup T$ is also a covering of A by intervals.

   b) Show that $\sigma(S \cup T) \leq \sigma(S) + \sigma(T)$.

3. (Chapter 1, Problem 4) Show that for $c \in \mathbb{R}$ and fixed k, the set (known as a hyperplane in $\mathbb{R}^n$)

$$A = \{x = (x_1, x_2, \ldots, x_k, \ldots, x_n) \in \mathbb{R}^n \mid x_k = c\}$$

has Lebesgue outer measure 0.

4. (Chapter 1, Problem 5) Suppose A and B are both Lebesgue measurable. Prove that if both A and B have measure zero, then $A \cup B$ is Lebesgue measurable and $m(A \cup B) = 0$.

    a) Do this directly from the definition.

    b) Give a shorter proof by using Theorem $1.2.5$.

5. (Chapter 1, Problem 6) Suppose A has Lebesgue measure zero and $B \subseteq A$. Prove B is Lebesgue measurable and $m(B) = 0$.

6. (Chapter 1, Problem 11) Let A be a subset of $\mathbb{R}$ and $c \in \mathbb{R}$. Define $A+c$ to be the set

$$A+c = \{x+c \mid x \in A\}$$

a) Prove $m^*(A+c) = m^*(A)$.

b) Prove that $A+c$ is Lebesgue measurable if and only if A is Lebesgue measurable.