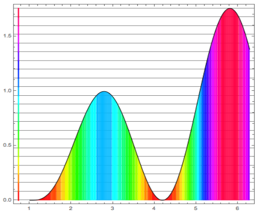
UC RIVERSIDE
Department of Mathematics

MATH 151C - Advanced Calculus



Homework 1

Due date: Thursday, April 29 at 11:59pm

Instructions:

- You are encourage to work in group but you have to write your individual solution
- You can use any book, article or web-based mathematical material or computational software
- You can ask Ryan, or Estela questions
- Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
- The problems from the textbook will be indicated by the chapter and corresponding number.
- The homework needs to be typeset in “LaTeX” .

1. (Chapter 1, Problem 6) Let E be a measurable subset of $\mathbb{R}^n$. Show that given $\varepsilon > 0$ there is a closed set F and an open set G with $F \subseteq E \subseteq G$ and $m(G \backslash E) < \varepsilon$.

2. (Chapter 1, Problem 24) Let A be a subset of $\mathbb{R}^n$. Show that there is a set H of type G_δ so that

$$A \subseteq H \text{ and } m^*(A) = m^*(H) \; .$$

3. (Chapter 2, Problem 1) Let $E \subseteq [a,b]$ and let $\mathcal{X}_E$ be the characteristic function of E. Prove that $\mathcal{X}_E(x)$ is a measurable function if and only if E is a measurable set.

4. (Chapter 2, Problem 3) Let $[c,d] \subseteq [a,b]$. Show that if f is measurable on $[a,b]$, then f is measurable on $[c,d]$.

5. (Chapter 2, Problem 4) Find an example of a pointwise bounded sequence of measurable functions $\{f_n\}$ on $[0,1]$ such that each $f_n(x)$ is a bounded function but $f^*(x) = \limsup_{n\to\infty} f_n(x)$ is not a bounded function.Suppose A has Lebesgue measure zero and $B \subseteq A$. Prove B is Lebesgue measurable and $m(B) = 0$.

6. (Chapter 2, Problem 8) Suppose f is measurable on $I = [a,b]$ and $f(x) \geq 0$ a.e. on I. Prove that if the set $\{x \in I \mid f(x) > 0\}$ has positive measure, then for some positive integer n the set

$$E_n = \left\{ x \in I \mid f(x) > \frac{1}{n} \right\}$$

has positive measure.