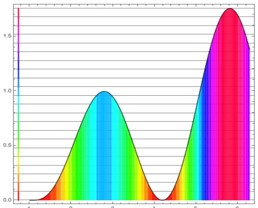
UC RIVERSIDE
Department of Mathematics

MATH 151C - Advanced Calculus



Homework 1

Due date: Thursday, May 6 at 11:59pm

Instructions:

* You are encourage to work in group but you have to write your individual solution
* You can use any book, article or web-based mathematical material or computational software
* You can ask Ryan, or Estela questions
* Chegg, Math Stack Exchange, or any other source where you can copy solutions is not allowed
* The problems from the textbook will be indicated by the chapter and corresponding number.
* The homework needs to be typeset in "LaTeX" .

1. (Chapter 2, Problem 13) Let f and g be bounded, Lebesgue integrable functions on $[a,b]$. Show that $f+g$ is Lebesgue integrable on $[a,b]$ and

$$\int_a^b (f+g) = \int_a^b f + \int_a^b g.$$

Hint: Exercise 11 might be useful.

2. (Chapter 2, Problem 14) Let h be a bounded function that is zero a.e. in $[a,b]$. Show that h is Lebesgue integrable on $[a,b]$ and

$$\int_a^b h = 0.$$

3. (Chapter 2, Problem 15) Let φ be a simple function defined on $[a,b]$.

   a) Show that φ is measurable on $[a,b]$.

   b) Show that φ is Lebesgue integrable on $[a,b]$. Use the definition of the Lebesgue integral to compute $\int_a^b \varphi$.

4. (Chapter 2, Problem 16) Let $f \in \mathcal{L}[a,b]$. Show that if g is a bounded measurable function, then $fg \in \mathcal{L}[a,b]$.

5. (Chapter 2, Problem 17) Prove or give a counterexample: If $f, g \in \mathcal{L}[a,b]$, then $fg \in \mathcal{L}[a,b]$.

6. (Chapter 2, Problem 19) Let $f \in \mathcal{L}[a,b]$ and A and B be measurable subsets of $[a,b]$.

   a) If $A \cap B = \emptyset$, show that
   $$\int_{A \cup B} f = \int_A f + \int_B f$$

   b) State and prove a result for the case that $A \cap B \neq \emptyset$.

   c) What can you conclude if $A = [a,c]$ and $B = [c,b]$ for some $c \in (a,b)$?