Math 10A Final Exam Winter 2006

Guan

November 17, 2010

Your pledge: I pledge my honor that I have not violated the honor rule and done anything related to cheating during this examination.

Math 10A Final Exam

- This is a close book exam. The total points are 100+5.
- In each problem, you have to show every step of your calculation.

Name:____________________________

ID Number:________________________

1. (10 points) Find the first and the second order partial derivatives:

   (a) $f(x, y) = e^{xy} + y^{10}$.

   (b) $f(x, y, z) = x(z^2 - e^y)^{10}$.

2. (10 points) Are following functions continuous at $(0,0)$? Explain your reasons.

   (a) $f(x,y) = xy^{10} + y\sin^{10} 2x$.

   (b)
$$f(x,y) = \begin{cases} \frac{x^2+y^2}{2006x^2+y^2} & \text{if} \quad (x,y) \neq (0,0) \\ \frac{2}{2007} & \text{if} \quad (x,y) = (0,0). \end{cases}$$

   (c)
$$f(x,y) = \begin{cases} \frac{x^{2006}+y^{10}}{x^2+y^2} & \text{if} \quad (x,y) \neq (0,0) \\ 1 & \text{if} \quad (x,y) = (0,0). \end{cases}$$

3. (10 points) Find the derivative matrix $Df(x, y)$ for $f(u, v) = (u^2v, u + v^2)$, $u = \sin(x + y)$, $v = xy^2$.

4. (10 points) Find a normal vector and the tangent plane of the surface $e^z + x - y \sin z = 1$ at point $(0, 1, 0)$.

5. (10 points) Find the maximum of the function $f(x, y) = x^2 - y^2$ on the domain $9x^2 + y^2 \leq 10$.

6. (10 points) Find the second order Taylor series of the function $f(x, y) = e^y + x \sin y$ at $(0, 0)$,

7. (10 points) Find all the critical points of $f(x, y) = \cos(x + y) + y^3$ and use the second derivatives to test the nature of them. Find the local maximal and minimal points.

8. (10 points) Assume the acceleration $a(t) = (\sin t, 10t, t^2 - 1)$ and the initial position $c(0) = (10, 0, 0)$. Find the position curve $c(t)$ if the initial velocity is $v(0) = (0, 0, 2006)$.

9. (10 points) Find the arc length of the curve $c(t) = (13(t - \sin t), 12(1 - \cos t), 5(1 - \cos t))$ with $t \in [0, 2\pi]$.

10. (10+5 points) Find the divergence of the vector field $F = (xyz, x^3, y^3)$. Is F a curl of a vector field? (bonus problem) Is F a gradient of a function?