## Sample Problems for the Second Exam

- *The exam is on Thursday, 11/30, 2:10 pm – 3:30 pm.*
- *In each problem, you have to show every step of your calculation.*

**1.** Decide whether the following pairs of functions are linearly independent or not:

(1) $\sin(\pi + x)$ and $\sin x$;
(2) $e^x$ and $xe^x$.

**2.** Solve the following second order linear equations:

(1) $2y'' - 3y' + y = 0$;
(2) $y'' - y = 8xe^x$;
(3) $y'' - 4y' + 3y = 20\cos x$.

**3.** Solve the following initial value problems:

(1) $y'' - 4y' + 4y = 0, \qquad y(0) = 1, y'(0) = 1$;
(2) $y'' + 2y' + 2y = 0, \qquad y(0) = 1, y'(0) = 0$;
(3) $y'' + y' - 2y = 2x, \qquad y(0) = 0, y'(0) = 1$.

**4.** Knowing that $y = x$ and $y = x^{-1}$ are two solutions of the differential equation

$$x^2y'' + xy' - y = 0,$$

(1) find the solution of that equation satisfying $y(1) = 0$ and $y'(1) = 2$;
(2) find the solutions of the nonhomogeneous equation

$$x^2y'' + xy' - y = x^{-2}.$$

**5.** Knowing that $y = x$ is a solution of

$$f(x)y'' + xg(x)y' - g(x)y = 0.$$

(1) Find all the solutions.
(2) If $g(x) = 1$ and $f(x) = x^2$, find all the solutions.

**6.** A 32 lb weight is attached to a frictionless spring, which in turn is suspended from the ceiling. The weight stretches the spring 2 ft and comes to a rest in its equilibrium position. The weight is then pulled down an additional 6 inches and released.

(1) Find the resulting motion of the weight as a function of of time.
(2) Find the amplitude, phase angle, period and frequency of the resulting motion.
(3) At what time does the weight first pass through the equilibrium position and what is the velocity at that time?
(4) Sketch the graph of the motion of the weight.
(5) Suppose the above spring-weight system is subject to a damping force (e.g., underwater) of 4 lb ft/sec, determine the motion of weight.
(6) Is the damped system overdamped, underdamped, or critical damped?
(7) sketch the graph of the motion of the weight.